On de Sitter Invariant Special Relativity and Cosmological Constant as Origin of Inertia

Han-Ying Guo[1,2,3],[*] Chao-Guang Huang[2,3],[†] Yu Tian[3],[‡] Zhan Xu[4],[§] and Bin Zhou[2,3],[¶]

[1] *China Center of Advanced Science and Technology (World Lab.),*

P.O. Box 8730, Beijing 100080, China,

[2] *Institute of High Energy Physics, Chinese Academy of Sciences,*

P.O. Box 918-4, Beijing 100039, China,

[3] *Institute of Theoretical Physics, Chinese Academy of Sciences,*

P.O.Box 2735, Beijing 100080, China, and

[4] *Physics Department, Tsinghua University, Beijing 100084, China.*

(Dated: June 17, 2004)

Weakening the Euclidean assumption in special relativity and the coordinate-independence hypothesis in general relativity for the de Sitter space, we propose a de Sitter invariant special relativity with two universal constants of speed c and length R based on the principle of relativity and the postulate of universal constants c and R on de Sitter space with Beltrami metric. We also propose a postulate on the origin of the inertial motions and inertial systems as a base of the principle of relativity. We show that the Beltrami-de Sitter space provides such a model that the origin of inertia should be determined by the cosmological constant Λ if the length R is linked with Λ. In addition, via the 'gnomonic' projection the uniform straight-line motion on Beltrami-de Sitter space is linked with the uniform motion along a great 'circle' on de Sitter space embedded in 5-d Minkowski space.

PACS numbers: 04.20.Cv, 03.30.+p, 98.80.Jk, 02.40.Dr.

I. INTRODUCTION

Recent observations show that our universe is accelerated expanding, asymptotic de Sitter (dS) with a positive cosmological constant Λ [1], [2]. However, there are lots of puzzles related to the dS space. How to introduce observables? What is the statistical origin of the entropy of cosmological horizon? How to more consistently connect the physical laws

in laboratory scale and in cosmic scale with asymptotic dS? There is no way to get dS space from string theory or M-theory so far, how to rescue? And so on so forth. These present exciting challenges at both theoretical and experimental levels and may touch the foundation of physics [3].

In special relativity ($\mathcal{SR}_c$), one of the breakthroughs in the last century and of the cornerstones of modern physics, in addition to the fundamental principles, it is assumed that the 3-d space (without gravity) is flat as the same as the 3-d space (without gravity) in Newton mechanics [4]. Namely, it is Euclidean. In the large scale of free space, it is less observation base and should be weakened. In general relativity ($\mathcal{GR}$), another breakthrough and milestone, the 4-d spacetime is assumed to be not flat in general and the Λ term is put in by hand. In addition, if one would take the usual understanding in $\mathcal{GR}$ that physics should completely be coordinate-independent, it might lead to the above puzzles and difficulties about dS and asymptotically dS .

On the other hand, as the base of the principle of relativity (PoR) in both Newtonian mechanics and $\mathcal{SR}_c$ the origin of the inertial motions and inertial coordinate systems, the origin of inertia for short, together with the origin of inertial mass are two related but different long-standing problems. We should also distinguish the origin of inertia from the origin of the local inertia in $\mathcal{GR}$, i.e. the local inertial systems in the local Minkowski space and the local inertial motions along the geodesics, respectively. It is an issue of the principle of equivalence.

As was pointed out [5], in fact, the PoR in the theory of relativity and the cosmological principle in modern cosmology seem to be in conflict. Thus, logically it seems also hard to explain the origin of inertia in the theory of relativity, since the all distant stars as a whole should roughly fit the cosmological principle. The origin of local inertia seems also the case [6]. Thus, the origin of inertia seems still open.

In this paper, we would weaken the Euclidean assumption in $\mathcal{SR}_c$ and the coordinate-independence hypothesis in $\mathcal{GR}$ for the dS space to explore what could happen to it. In addition, based on recent observations on the dark matter and dark energy or the cosmological constant as its simplest form, we present a restatement on the origin of the both inertia and local inertia and propose a postulate on the origin of inertia for the case without any matter and dark matter. It is then closely related to the cosmological principle. It turns out that the $\mathcal{SR}_c$ can be generalized to a special relativity of dS invariance with an additional universal constant of length R, $\mathcal{SR}_{c,R}$. And it provides such a model that the cosmological constant is just the origin of inertia on dS space via an interesting relation between the PoR and the cosmological principle on it, if the length R is linked with Λ. The analogues approach to the anti-de Sitter (AdS) space can also work.

The $\mathcal{SR}_{c,R}$ is based on the PoR and the postulate of invariant universal signal speed c and length R (PoI$_{c,R}$). If R is taken as $R^2 = 3\Lambda^{-1}$, the cosmological constant appears at principle level and $\mathcal{SR}_{c,R}$ may also be denoted as $\mathcal{SR}_{c,\Lambda}$. Thus, unlike $\mathcal{SR}_c$ where there should be no room for Λ and unlike $\mathcal{GR}$ where there should be no inertial motion with

uniform coordinate velocity, there are proper rooms for Λ and a kind of inertial motions with uniform coordinate velocities along straight lines in the curved spacetimes of constant curvature. We may define a set of the observable of free particles and generalize famous Einstein's energy-momentum-mass formula. We may also define two kinds of simultaneity. The first is directly from PoR and for the experiments in local laboratories. The second is for the cosmological observations. It leads to an empty accelerated expending cosmological model with 3-d space of positive curvature in the order Λ. This is an important prediction. Even such a dS space is empty, but our universe might be so asymptotically. This is already roughly indicated by $\Omega_T = 1.02 \pm 0.02$ from WMAP [2].

These are in fact very important properties of dS space [7],[8],[9], which are ignored for long time in $\mathcal{GR}$, might be due to the coordinate-independent hypothesis approach. Namely, among various metrics of dS spaces, there is an important one in which dS space is in analog with Minkowski space. It is the dS space with Beltrami-like metric, called the Beltrami-de Sitter (BdS) space. It is precisely the Beltrami-like model [10] of a 4-hyperboloid $\mathcal{S}_R$ in 5-d Minkowski space, $BdS \simeq \mathcal{S}_R$. In BdS space there exist a set of Beltrami coordinate systems covering BdS patch by patch, and in which particles and light signals move along the timelike and null geodesics, respectively, with *constant* coordinate velocities. Therefore, they look like in free motion in a space without gravity. Thus, the Beltrami coordinates and observers at these systems should be regarded as of inertia.

There properties can also be seen as follows. If we start with the 4-d Euclid geometry and weaken the fifth axiom, then there exist 4-d Riemann, Euclid, and Lobachevski geometries at almost equal footing. For the non-Euclid ones, geodesics are in one-to-one correspondence with straight lines in Beltrami coordinate systems, which are generalizations of the coordinate systems in Beltrami model of Lobachevski plane [10], and under corresponding transformation groups the systems transform among themselves. Now changing the metric signature to -2, these non-Euclid constant curvature spaces turn to dS/AdS spaces with the Beltrami metrics, the Euclid one to Minkowski space M, and those straight lines are classified by timelike, null and spacelike straight world-lines, respectively. In addition, from projective geometry with an antipodal identification of, say, dS space $dS/Z_2 \subset RP^4$, it is also the case except this leading to non-orientability [7],[8],[11]. More concretely, one may take the 'gnomonic' projection, which is called sometimes the 'circle-rectilinear' mapping. It maps the great 'circle' on dS/AdS as pseudo-spheres embedded in 5-d Minkowski spaces to the straight lines on the dS/AdS spaces with the Beltrami metric. Of course, in order to preserve the orientation, the antipodal identification should not be taken.

Thus, in analog with $\mathcal{SR}_c$, on BdS space, say, there should exist such motions and observers of inertia. And PoR should also be available on it. In addition to the invariant universal speed, the speed of light c, there is another invariant universal length R as the curvature radii, so the postulate of invariant of velocity of light should be replaced by the PoI$_{c,R}$. That is why based on these two principles, the dS invariant special relativity $\mathcal{SR}_{c,R}$ can be set up.

Furthermore, in view of the $\mathcal{SR}_{c,R}$, we show that the BdS space provides such a model, in which the origin of inertia should be determined by Λ. In addition, via the 'gnomonic' projection there is a relation between the inertial motion on BdS space and the uniform motion along a great 'circle' on dS space embedded in a 5-dimensional Minkowski space $M^{1,4}$.

This paper is arranged as follows. In the sections 2-4, we set up a framework for $\mathcal{SR}_{c,R}$ based on the two postulates [7, 8]. We consider how to introduce a set of observable for the particles and signals, generalize Einstein's famous formula and define the intervals, light cone and two kinds of simultaneity. We show that the 3-cosmic space in the BdS space is a slightly closed. In the section 5, we present a restatement on the origin of both inertia and local inertia based on recent observations, propose the postulate on the origin of inertia for the case without both matter and dark matter and show that the BdS space is just such a model of Λ as the origin of inertia on it. Finally, we end with a few remarks.

II.   DE SITTER INVARIANT SPECIAL RELATIVITY

We now introduce the two postulates and set up a framework for $\mathcal{SR}_{c,R}$ [7, 8].

The PoR requires *there exist a set of inertial reference frames, in which the free particles and light signals move with uniform straight lines, the laws of nature without gravity are invariant under the transformations among them.* The PoI$_{c,R}$ requires *there exist two invariant universal constants — speed c and length R.*

Owing to Umov, Weyl and Fock [12], it can be proved that the most general form of the transformations among inertial coordinate systems F and F'

$$x'^i = f^i(x^i), \quad x^0 = ct, \quad i = 0, \cdots, 3, \tag{2.1}$$

which transform a uniform straight line motion in F to a motion of the same nature in F' are that *the four functions f^i are ratios of linear functions, all with the same denominators.*

As in $\mathcal{SR}_c$, the PoR implicates that the metric, if it exists, on inertial frame in spacetime is of signature ± 2 and invariant under a transformation group with ten parameters including space-time 'translations' (4), boosts (3) and space rotations (3), respectively. Thus, due to maximally symmetric space theory [6], the necessary and sufficient condition for 4-d spaces with invariant metric of signature ± 2 under ten-parameter transformation group is that they are $dS/M/AdS$ of positive, zero, or negative constant curvature, invariant under group $SO(1,4)$, $ISO(1,3)$ or $SO(2,3)$, respectively. The PoI$_{c,R}$ requires that there are proper rooms for the constant c as in transformations (2.1) and the invariant length R, which should be the curvature radii of dS/AdS, respectively.

We now focus on the dS space. It can be regarded as a 4-d hyperboloid $\mathcal{S}_R$ embedded in

a 5-d Minkowski space with $\eta_{AB} = \text{diag}(1, -1, -1, -1, -1)$:

$$\mathcal{S}_R: \quad \eta_{AB}\xi^A\xi^B = -R^2, \tag{2.2}$$

$$ds^2 = \eta_{AB}d\xi^A d\xi^B, \tag{2.3}$$

where $A, B = 0, \ldots, 4$. Clearly, Eqs. (2.2) and (2.3) are invariant under dS group $\mathcal{G}_R = SO(1,4)$. Via the 'gnomonic' projection, $\mathcal{S}_R$ becomes the BdS space, in which there exist Beltrami coordinates [8] defined patch by patch. For intrinsic geometry of $BdS \simeq \mathcal{S}_R$ space, there are at least eight patches $U_{\pm\alpha} := \{\xi \in \mathcal{S}_R : \xi^\alpha \gtrless 0\}, \alpha = 1, \cdots, 4$. In $U_{\pm 4}$, for instance, the Beltrami coordinates are

$$x^i|_{U_{\pm 4}} = R\xi^i/\xi^4, \qquad i = 0, \cdots, 3; \tag{2.4}$$

$$\xi^4|_{U_{\pm 4}} = (\xi^{0^2} - \sum_{a=1}^{3}\xi^{a2} + R^2)^{1/2} \gtrless 0, \tag{2.5}$$

which are like locally the inhomogeneous coordinates in projective geometry but *without the antipodal identification.* In the patches $U_{\pm a}$, $a = 1, 2, 3$,

$$y^{j'}|_{U_{\pm a}} = R\xi^{j'}/\xi^a, \quad j' = 0, \cdots, \hat{a} \cdots, 4; \quad \xi^a \neq 0, \tag{2.6}$$

where $\hat{a}$ means omission of a. It is important that all transition functions in intersections are of $\mathcal{G}_R$ in the type (2.1). For example, in $U_4 \bigcap U_3$, the transition function $T_{4,3} = \xi^3/\xi^4 = x^3/R = R/y^4 \in \mathcal{G}_R$ so that $x^i = T_{4,3}y^{i'}$.

In each patch, there are condition and Beltrami metric

$$\sigma(x) = \sigma(x,x) := 1 - R^{-2}\eta_{ij}x^i x^j > 0, \tag{2.7}$$

$$ds^2 = [\eta_{ij}\sigma(x)^{-1} + R^{-2}\eta_{ik}\eta_{jl}x^k x^l \sigma(x)^{-2}]dx^i dx^j. \tag{2.8}$$

Under fractional linear transformations of type (2.1), which are transitive in $\mathcal{G}_R$ sending $A(a^i)$ to the origin,

$$x^i \to \tilde{x}^i = \pm\sigma(a)^{1/2}\sigma(a,x)^{-1}(x^j - a^j)D^i_j,$$
$$D^i_j = L^i_j + R^{-2}\eta_{jk}a^k a^l(\sigma(a) + \sigma(a)^{1/2})^{-1}L^i_l, \tag{2.9}$$
$$L := (L^i_j)_{i,j=0,\cdots,3} \in SO(1,3),$$

where $\eta_{ij} = \text{diag}(1, -1, -1, -1)$ in $U_{\pm\alpha}$, *Eqs. (2.7) and (2.8) are invariant. The inertial frames and inertial motions transform among themselves, respectively.* Note that Eqs. (2.7)-(2.9) are defined on patch by patch. $\sigma(x) = 0$ is the projective boundary of BdS, denoted by $\partial(BdS)$.

For two separate events $A(a^i)$ and $X(x^i)$ in BdS,

$$\Delta_R^2(a,x) = R^2[\sigma^{-1}(a)\sigma^{-1}(x)\sigma^2(a,x) - 1] \tag{2.10}$$

is invariant under $\mathcal{G}_R$. Thus, the interval between A and B is timelike, null, or spacelike, respectively, according to

$$\Delta_R^2(a,b) \gtreqless 0. \tag{2.11}$$

The proper length of timelike or spacelike between A and B are integral of $\mathcal{I}ds$ over the geodesic segment $\overline{AB}$:

$$S_{t-like}(a,b) = R\sinh^{-1}(|\Delta(a,b)|/R), \tag{2.12}$$
$$S_{s-like}(a,b) = R\arcsin(|\Delta(a,b)|/R), \tag{2.13}$$

where $\mathcal{I} = 1, -i$ for timelike or spacelike, respectively. Note that there exist such kind of pairs (A, B) that there is no geodesic segment $\overline{AB}$ to connect them. We will explain this issue elsewhere.

The light-cone at A with running points X is

$$\mathcal{F}_R := R\{\sigma(a,x) \mp [\sigma(a)\sigma(x)]^{1/2}\} = 0. \tag{2.14}$$

It satisfies the null-hypersurface condition. At the origin $a^i = 0$, the light cone becomes a Minkowski one and c is numerically the speed of light in the vacuum.

III. INERTIAL MOTION, OBSERVABLES AND BELTRAMI SIMULTANEITY

It is easy to check that on BdS space the geodesics are Lobachevski-like straight lines. In fact, the 'gnomonic' projection provides such an intuitive picture.

A particle with mass m_Λ moves along a timelike geodesic, which has the first integration

$$\frac{dp^i}{ds} = 0, \quad p^i := m_\Lambda \sigma(x)^{-1}\frac{dx^i}{ds} = C^i = \text{const.} \tag{3.1}$$

This implies under the initial condition

$$x^i(s=0) = b^i, \qquad \frac{dx^i}{ds}(s=0) = c^i$$

with the constraint $g_{ij}(b)c^ic^j = 1$, the geodesic is just a straight world-line

$$x^i(w) = c^iw + b^i, \tag{3.2}$$

where $w = w(s)$ is given by

$$w(s) = \begin{cases} Re^{\mp s/R}\sinh\frac{s}{R}, & \eta_{ij}c^ic^j = 0, \\ \dfrac{R\sinh\frac{s}{R}}{\dfrac{\eta_{ij}c^ib^j}{R\sigma(b)}\sinh\frac{s}{R} + \cosh\frac{s}{R}}, & \eta_{ij}c^ic^j \neq 0. \end{cases} \tag{3.3}$$

Similarly, a light signal moves inertially along a null geodesic, which still has the first integration

$$\sigma^{-1}(x)\frac{dx^i}{d\lambda} = \text{constant}, \qquad (3.4)$$

where λ is an affine parameter. Under the constraint $g_{ij}(b)\,c^i c^j = 0$ and initial condition

$$x^i(\lambda = 0) = b^i, \qquad \frac{dx^i}{d\lambda}(\lambda = 0) = c^i, \qquad (3.5)$$

the null geodesic can be expressed as a straight line

$$x^i = c^i w(\lambda) + b^i, \qquad (3.6)$$

where

$$w(\tau) = \begin{cases} \lambda, & \eta_{ij}\,c^i c^j = 0, \\ -\dfrac{R^2\sigma(b)}{|\eta_{ij}c^i c^j|}\left(\dfrac{1}{\lambda + \lambda_0} - \dfrac{1}{\lambda_0}\right), & \eta_{ij}\,c^i c^j \neq 0, \end{cases} \qquad (3.7)$$

with

$$\lambda_0 = \sqrt{\frac{R^2\sigma(b)}{|\eta_{ij}c^i c^j|}}.$$

Thus, the motions of both free particles and light signals are indeed inertia, i.e. the coordinate velocity components are constants, respectively:

$$\frac{dx^a}{dt} = v^a; \quad \frac{d^2 x^a}{dt^2} = 0; \quad a = 1, 2, 3. \qquad (3.8)$$

Note that these properties are well defined patch by patch. Conversely, all straight lines are geodesics.

In $\mathcal{GR}$, however, these properties are ignored might be due to the coordinate-independence hypothesis.

Now we define sets of the observable for free particles and signals. From (3.1), it is natural to define the conservative quantities p^i as the 4-momentum of a free particle with mass m_Λ and its zeroth component as the energy. Note that it is no longer a 4-vector rather a pseudo 4-vector. Furthermore, for the particle a set of conserved quantities L^{ij} may also be defined

$$L^{ij} = x^i p^j - x^j p^i; \quad \frac{dL^{ij}}{ds} = 0. \qquad (3.9)$$

These may be called the 4-angular-momentum, which is a pseudo anti-symmetric tensor. Further, p^i and L^{ij} constitute a conserved 5-d angular momentum in $\mathcal{S}_R$

$$\mathcal{L}^{AB} := m_\Lambda(\xi^A\frac{d\xi^B}{ds} - \xi^B\frac{d\xi^A}{ds}); \quad \frac{d\mathcal{L}^{AB}}{ds} = 0. \qquad (3.10)$$

Thus, (3.10) defines *a kind of the uniform motions along great 'circles' hidden on $\mathcal{S}_\Lambda$ that related to the inertial motions on BdS space via the 'gnomonic' projection without antipodal identifications and vice versa.*

For such a kind of free particles, there is a *generalized Einstein's formula including 4-angular momentum*:

$$-\frac{1}{2R^2}\mathcal{L}^{AB}\mathcal{L}_{AB} = E^2 - p_b p^b - \frac{1}{2R^2}L^{ij}L_{ij} = m_\Lambda^2, \tag{3.11}$$

where $\mathcal{L}_{AB} = \eta_{AC}\eta_{BD}\mathcal{L}^{CD}$, $p_a = \delta_{ab}p^b$, $L_{ij} = \eta_{ik}\eta_{jl}L^{kl}$. Here m_Λ the *dS invariant inertial mass for a free particle* well-defined along with the energy, momentum, boost and angular momentum at classical level. Further, m_Λ^2 is the eigenvalue of the first Casimir operator of the dS algebra. Thus, $\mathcal{SR}_{c,R}$ offers *a dS invariant definition and classification of the inertial mass*. In addition, spin can also be well defined and related to the eigenvalue of the second Casimir operator. In fact, the generators forming an $so(1,4)$ algebra of (2.9) read in BdS space

$$\begin{aligned}
\mathbf{P}_i &= (\delta_i^j - R^{-2}x_i x^j)\partial_j, \quad x_i := \eta_{ij}x^j, \\
\mathbf{L}_{ij} &= x_i \mathbf{P}_j - x_j \mathbf{P}_i = x_i \partial_j - x_j \partial_i \in so(1,3),
\end{aligned} \tag{3.12}$$

with two Casimir operators

$$\mathbf{C}_1 = \mathbf{P}_i \mathbf{P}^i - \frac{1}{2}R^{-2}\mathbf{L}_{ij}\mathbf{L}^{ij}, \tag{3.13}$$

$$\mathbf{C}_2 = \mathbf{S}_i \mathbf{S}^i - R^{-2}\mathbf{W}^2, \tag{3.14}$$

where $\mathbf{P}^i = \eta^{ij}\mathbf{P}_j$, $\mathbf{L}^{ij} = \eta^{ik}\eta^{jl}\mathbf{L}_{kl}$, $\mathbf{S}_i = \frac{1}{2}\epsilon_{ijkl}\mathbf{P}^j\mathbf{L}^{kl}$, $\mathbf{S}^i = \eta^{ij}\mathbf{S}_j$, $\mathbf{W} = \frac{1}{8}\epsilon_{ijkl}\mathbf{L}^{ij}\mathbf{L}^{kl}$.

Thus, $\mathcal{SR}_{c,R}$ offers a consistent way to define a set of the observable for free particles. These issues significantly confirm that the motion of a free particle is inertial, the Beltrami coordinate systems and corresponding observer of the system are all inertial as well.

As in $\mathcal{SR}_c$, in order to make sense of inertial motions and observables practically, one should define simultaneity and take space-time measurements. In $\mathcal{SR}_c$, due to the PoR, Minkowski coordinates have measurement significance. Namely, the difference in time coordinate stands for the time interval, and the difference in spatial coordinate stands for the spatial distance. Similar to $\mathcal{SR}_c$, one can define that two events A and B are simultaneous if and only if the Beltrami time coordinate x^0 for the two events are same,

$$a^0 := x^0(A) = x^0(B) =: b^0. \tag{3.15}$$

It is called the *Beltrami simultaneity* and with respect to it that free particles move inertially. The Beltrami simultaneity defines a 3+1 decomposition of spacetime

$$ds^2 = N^2(dx^0)^2 - h_{ab}\left(dx^a + N^a dx^0\right)\left(dx^b + N^b dx^0\right) \tag{3.16}$$

with the lapse function, shift vector, and induced 3-geometry on 3-hypersurface Σ_c in one coordinate patch.

$$\begin{aligned}
N &= \{\sigma_{\Sigma_c}(x)[1 - (x^0/R)^2]\}^{-1/2}, \\
N^a &= x^0 x^a [R^2 - (x^0)^2]^{-1}, \\
h_{ab} &= \delta_{ab}\sigma_{\Sigma_c}^{-1}(x) - [R\sigma_{\Sigma_c}(x)]^{-2}\delta_{ac}\delta_{bd}x^c x^d,
\end{aligned} \tag{3.17}$$

respectively, where $\sigma_{\Sigma_c}(x) = 1 - (x^0/R)^2 + \delta_{ab}x^a x^b/R^2$, δ_{ab} is the Kronecker δ-symbol, $a, b = 1, 2, 3$. In particular, at $x^0 = 0$, $\sigma_{\Sigma_c}(x) = 1 + \delta_{ab}x^a x^b/R^2$. In a vicinity of the origin of Beltrami coordinate system in one patch, 3-hypersurface Σ_c is isomorphic to a 3-sphere. For the $x^0 \neq 0$, it is also the case.

The Beltrami simultaneity defines the laboratory time in one patch. In the spirit of $\mathcal{SR}_c$ and due to the PoR in $\mathcal{SR}_{c,R}$, there are definite relations between the Beltrami coordinates and the standard clocks and rulers in laboratory in such a manner that measure the time of a process or the size of an object, one may just need to compare with Beltrami coordinates together with their relations with the standard clocks and rulers.

IV. PROPER-TIME SIMULTANEITY AND SLIGHTLY CLOSED COSMIC SPACE

It should be emphasized that there is another simultaneity in $\mathcal{SR}_{c,R}$ and it is directly related to the cosmological principle. It is *proper-time simultaneity* with respect to a clock rest at spatial origin of the Beltrami coordinate system.

The proper time $\tau = \tau_\Lambda$ of a rest clock on the time axis of Beltrami coordinate system, $\{x^a = 0\}$, reads

$$\tau = \tau_\Lambda = R \sinh^{-1}(R^{-1}\sigma^{-1/2}(x)x^0). \tag{4.1}$$

Therefore, we can define that the events are simultaneous with respect to it, then these events are co-moving with the clock, if and only if

$$x^0 \sigma^{-1/2}(x, x) = \xi^0 := R \sinh(R^{-1}\tau) = \text{constant}. \tag{4.2}$$

The line-element on the simultaneous 3-d hypersurface, denoted by Σ_τ, can be defined as

$$dl^2 = -ds_{\Sigma_\tau}^2, \tag{4.3}$$

where

$$
\begin{aligned}
&ds_{\Sigma_\tau}^2 = R_{\Sigma_\tau}^2 dl_{\Sigma_\tau 0}^2, \\
&R_{\Sigma_\tau}^2 := \sigma^{-1}(x, x)\sigma_{\Sigma_\tau}(x, x) = 1 + (\xi^0/R)^2, \\
&\sigma_{\Sigma_\tau}(x, x) := 1 + R^{-2}\delta_{ab}x^a x^b > 0, \\
&dl_{\Sigma_\tau 0}^2 := \{\delta_{ab}\sigma_{\Sigma_\tau}^{-1}(x) - [R\sigma_{\Sigma_\tau}(x)]^{-2}\delta_{ac}\delta_{bd}x^c x^d\}dx^a dx^b.
\end{aligned}
\tag{4.4}
$$

The relation of this simultaneity with the cosmological principle can be seen as follows. In fact, it is significant that if τ_Λ is taken as a "cosmic time", the Beltrami metric (2.8) becomes a Robertson-Walker-like metric with a positive spatial curvature and the simultaneity is globally defined in whole BdS space

$$ds^2 = d\tau^2 - dl^2 = d\tau^2 - \cosh^2(R^{-1}\tau)dl_{\Sigma_\tau 0}^2. \tag{4.5}$$

This shows that *the 3-d cosmic space is S^3 rather than flat. The deviation from the flatness is of order Λ.* Our universe should be asymptotically so.

The two definitions of simultaneity do make sense in different kinds of measurements. The first concerns the measurements in a laboratory and is related to the PoR and PoI$_{c,R}$ of $\mathcal{SR}_{c,R}$. The second concerns with cosmological effects. Furthermore, the relation between the Beltrami metric with coordinate time x^0 and its RW-like counterpart with cosmic time τ links the PoR and cosmological principle. It is very meaningful. The prediction that the spatial closeness of the universe asymptotically in the order of Λ is different from the standard cosmological model with flatness or with a free parameter k.

V. THE POSTULATE ON THE ORIGIN OF INERTIA AND Λ AS THE ORIGIN OF INERTIA IN $\mathcal{SR}_{c,R}$

Based upon recent observations, it is clear that in such a universe the problem of the origin of inertia and its local version should be restated in the sprit of Riemann, Mach and Einstein: *The origin of inertia and the origin of local inertia should be dominatingly determined by the dark matter and dark energy or the cosmological constant along with a little contribution from the distant stars and other regular matter and radiation.* Since all of them as a whole support the cosmological principle, if the statement might be true, it should be closely related to the cosmological principle except the cases that some individual star/matter is as a local gravitational source and so on. On the other hand, if this statement could make sense, for the empty dS space without either matter or dark matter, the Λ might contribute to the origin of inertia rather than its local version. Thus, we should also have such a postulate on inertia: *If in dS space there could exist the inertial motions and inertial systems, they should be determined by the cosmological constant.*

As was mentioned in the last sections, on the BdS space there is such a relation between the PoR and the cosmological principle. Thus, it indicates that BdS space may provide such a model on Λ as the origin of inertia. In fact, on BdS space two kinds of simultaneity just link the two principles. Namely, as was shown above, the Beltrami simultaneity is directly from the PoR for the experiments in the local laboratories. The proper time simultaneity is for the cosmological observations. If the proper time is taken as a cosmic time, it leads BdS space as an 'empty' accelerated expending cosmological model and 3-d space of positive curvature in the order Λ.

Importantly, it should be pointed out that the inertial motions and the inertial coordinate systems of the PoR on BdS space should be determined by Λ via the cosmological principle. What is needed to do is changing the timing from the cosmic time τ to the Beltrami time $x^0 = ct$ and vice versa. Namely, if the comoving observers on (4.5) would change the time measurement from the cosmic time τ to the Beltrami time x^0 according to the relation (4.1), they should become a kind of inertial observers and vice versa.

Thus, the $\mathcal{SR}_{c,R}$ on BdS space provides such a model that the origin of inertia is just Λ and it seems to be more complete and consistent in logic within the PoR, postulate on inertia and cosmological principle.

VI.   CONCLUSIONS

Weakening the Euclidean assumption in $\mathcal{SR}_c$ and the coordinate-independence hypothesis in $\mathcal{GR}$ for the dS space, we have set up the dS invariant special relativity with an invariant length R in addition to c, $\mathcal{SR}_{c,\Lambda>0}$. It is based on the PoR and PoI$_{c,R}$. Similarly, $\mathcal{SR}_{c,\Lambda<0}$ with AdS invariance can also be set up. The Beltrami coordinates, which are like locally inhomogeneous projective coordinates but without the antipodal identification, are inertial, the test particles and signals move inertially along the timelike, null straight world lines, respectively. There are also dS invariant definitions and classifications of the inertial mass and spin of the free particles and fields. Einstein's energy-momentum-mass formula is generalized. Intervals, light cone and two kinds of simultaneity are defined. There is also an interesting relation between the uniform motion along a great 'circle' hidden on $\mathcal{S}_\Lambda$ and the inertial motion on BdS space.

It is important the relation between the Beltrami metric and the RW-like metric. It links the PoR and cosmological principle, relates the coordinate time x^0 in the laboratory and the cosmic time τ in the cosmic scale. This also predicts that asymptotically the 3-d cosmic space is slightly closed in order of Λ. Hopefully, this prediction should be confirmed by the further data from WMAP in large scale.

We have also proposed a statement on the origin of all kinds of inertial motions and inertial systems based upon the recent observations on the dark sector in the cosmic scale and a postulate on the origin of inertia without any matter and dark matter for dS space. We show that it does make sense that the Λ provides the origin of inertia in the BdS space. In addition, it can also be shown that the Newton-Hooke space-times $\mathcal{NH}_\pm$ contracted from Beltrami-dS/AdS are also such kind of models with the origin of inertia.

Most properties here are in analog with $\mathcal{SR}_c$ except there is a proper room of R or Λ, which leads to those amazing aspects, and coincide with $\mathcal{SR}_c$ if $\Lambda \to 0$, i.e. the $\mathcal{SR}_c$ is $\mathcal{SR}_{c,\Lambda\to 0}$. However, all amazing aspects of the dS invariant $\mathcal{SR}_{c,R}$ disappear under such a limitation.

All local experiments would allow there might exist three theories of special relativity at almost equal footing, i.e. $\mathcal{SR}_{c,\Lambda>0}$, $\mathcal{SR}_{c,\Lambda=0}$, and $\mathcal{SR}_{c,\Lambda<0}$ with dS, Poincaré, and AdS invariance, respectively. Recent observations in cosmic scale show that there should be a positive cosmological constant. Therefore, the dS invariant special relativity, $\mathcal{SR}_{c,\Lambda>0}$, is more reasonable candidate for a starting point if one would modify and apply the relativistical physics to our expanding asymptotic dS universe as a whole in a more consistent manner.

In summery, if Galileo Galilei could arrange a large spaceship voyage, his friends might not need to shut themselves up 'in the main cabin below decks'[13]. During the voyage they could even carry on, with a sensitive microwave telescope and other instruments, the measurement of the spaceship drift with respect to the CMB and to explore how much of the Λ should contribute to the origin of inertial motion as well as to the origin of the hidden uniform motion along a great 'circle' in an asymptotic dS-cosmos.

Acknowledgments

[1] A.G. Riess et al, *Astro. J.* **116**, 1009(1998); S. Perlmutter et al,*Astrophys. J.* **517**, 565 (1999).

[2] C. L. Bennett et al, *Astrophys. J.* (Suppl.) **148** 1 (2003); M. Tegmark, et al, astro-ph/0310723.

[3] E. Witten, hep-th/0106109; A. Strominger, Talk at the String Satellite Conference to ICM 2002. August, 2002, Beijing.

[4] A. Einstein, *Annalen der Physik*, **17** (1905) 891.

[5] See, e.g. H. Bondi, *Observatory* (London),**82** (1962) 133; P.G. Bergmann, *Found. Phys.* **1** (1970) 17; N. Rosen, *Phys. Rev.* **D3** (1971) 2317.

[6] See, e.g., S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. New York: Wiley, 1972.

[7] The original idea is due to K.H. Look (Q.K. Lu), Why the Minkowski metric must be used? (1970) unpublished. For earlier works, see, K.H. Look, C.L. Tsou (Z.L. Zou) and H.Y. Kuo (H.Y. Guo), *Acta Physica Sinica* **23** (1974) 225; *Nature* (Shanghai, Suppl.), Modern Physics **1** (1980) 97; H.Y. Kuo, *Kexue Tongbao (Chinese Science Bulletin)* **22** (1977) 487 (all in Chinese); H.Y. Kuo, *Proceedings of the 2nd Marcel Grossmann Meeting on General Relativity*, ed. by R. Ruffini, (North-Holland Pub. Amsterdam, 1982) 801; H.Y. Guo, *Nucl. Phys. B* (Proc. Suppl.) **6** (1989) 381.

[8] H.-Y. Guo, C.-G. Huang, Z. Xu and B. Zhou, hep-th/0311156; H.-Y. Guo, C.-G. Huang, Y. Tian, Z. Xu and B. Zhou, hep-th/0403013, H.-Y. Guo, C.-G. Huang, Z. Xu and B. Zhou, hep-th/0403171; H.-Y. Guo, C.-G. Huang, and B. Zhou, hep-th/0404010.

[9] F. Gürsey, in *Relativity, Groups and Topology*, ed. C. DeWitt and B. DeWitt, (Blackie and Son Ltd, 1969).

[10] E. Beltrami, *Opere Mat.*, **1** (1868) 374-405.

[11] G.W. Gibbons, Private communication. March 2, 5, 2004.

[12] See, e.g., V. Fock, The Theory of Space Time and Gravitation, Pergamon Press (1964), and references therein.

[13] Galileo Galilei (1632), Dialogues Concerning the Two Chief World Systems, The Second Day. Trans. S. Drake, 2nd ed. 1967, U. California Press.